Dance, tangles, and topology!

The movement figures

The swap (S) movement corresponds to the operation +1

The rotation (R) movement corresponds to taking the negative reciprocal

The scheme for the mathematical operations

- Ropes untied (initial position) equals 0
- Doing the Swap corresponds to add 1 to the previous number (e.g. 2 + 1 = 3)
- Doing a rotation corresponds to the negative reciprocal, that is inverting and changing the sign of the previous number (e.g., the negative reciprocal of 3 is \(-\frac{1}{3}\))

The knot obtained thus corresponds to a rational number (in the previous example it corresponds to \(\frac{7}{3}\)). Now students have to search for a sequence of operations that eventually lead back to zero.

An example of a dance step sequence and the corresponding operations

Some knots and their associated fractions

*Image courtesy of the author*