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Exponential growth 1: learn the basics from confetti to understand pandemics

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Exponential growth has become part of daily life during the COVID-19 pandemic. These simple exercises help explain this tricky concept

If you find it difficult to explain the extremely rapid spread of the COVID-19 virus, you are not alone. The human imagination is not used to dealing with this kind of growth.

With these activities, your students will learn how to describe exponential growth, and how this applies to understanding the spread of infectious diseases. They are designed for students aged 11–13 and can be completed in 45 minutes.

Most people associate the term growth with linear growth, in which a quantity always increases by the same amount in a particular time period, independent of the initial value. For example, a human hair grows, on average, 0.4 mm per day, regardless of how long it is, so it is easy to calculate how much a hair has grown in 10 or 100 days.

It is different for financial investment, where the amount of interest you earn depends on the amount of money already in your account. If the increase is a function of the current quantity, it is called exponential growth. This kind of growth is somewhat counterintuitive; it is difficult for the human imagination to estimate what the increase will be after a certain time. However, this is an important concept to understand since it affects many real-world situations, and failing to grasp it can lead to poor decisions.

One important field in which exponential growth applies is epidemiology, where we speak of the reproduction number, *R*, which specifies how many people are infected, on average, by one infectious person.

Through these activities, students will learn about different types of exponential growth and how to classify some of them using the *R value*. In the context of the COVID-19 pandemic, they will learn how the spread of the virus can be slowed by taking measures that reduce the *R value*.

The calculations are best done using a calculator or an Excel spreadsheet. The worksheet contains tables and coordinate systems to complete.

Activity 1 – Confetti for a party

To introduce students to the complex topic of exponential growth and its mathematical description, first we look at a simple real-life example.

What is the quickest and easiest way to produce lots of confetti using a sheet of paper and a hole punch?

The easiest way is to fold the paper several times before punching it to produce several snippets of paper in one punch. But what is the relationship between the number of folds and the number of confetti you get from one punch?

Safety note

Please ensure that students fold the paper a maximum of five times, to avoid damaging the hole punch or injury.

Materials

- Paper; one sheet per student
- A hole punch (ideally one per student or group)
- The accompanying worksheet

Procedure

1. Tell students that, for a party, they should make as much confetti as quickly as possible from their sheet of paper by using a hole punch. They have three minutes to do this.
2. After three minutes, discuss with students their most effective approach. Many students will fold the paper several times before punching.
3. To discover the relationship between the number of folds and the number of confetti obtained, students need to complete the diagram with concentric circles on the worksheet. Each circle represents one paper fold. On each circle, have them draw the number of confetti they will obtain when they punch paper folded as many times as indicated on the circle. The diagram should look like this:

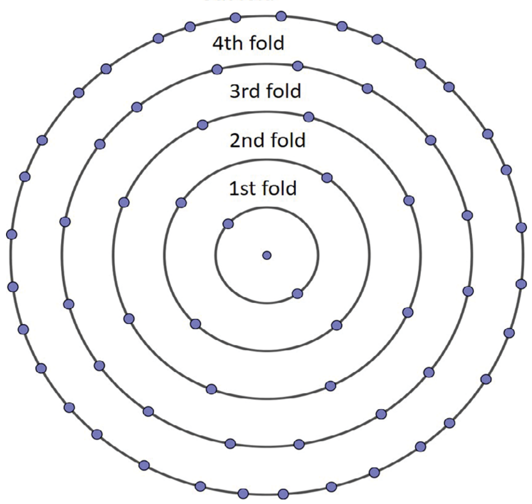


Image courtesy of Wolfgang Vieser

4. Ask students to track how the number of confetti might depend on the number of folds by filling out the second row of Table 1. The table should look like this:

Number of paper folds	0	1	2	3	4	5
Number of confetti obtained	1	2	4	8	16	32
Relationship						

Part 2: Finding a mathematical relationship and extending it to larger numbers

1. Have your students discuss in groups what relationship there might be between the values in the first and second rows of the table. Perhaps hint that it could have something to do with the number two.
2. Complete the table with the students' predictions for the connection between the first two rows of the table. The table should then look like this:

Number of paper folds	0	1	2	3	4	5
Number of confetti obtained	1	2	4	8	16	32
Relationship	2^0	2^1	$2 \times 2=2^2$	$2 \times 2 \times 2=2^3$	$2 \times 2 \times 2 \times 2=2^4$	$2 \times 2 \times 2 \times 2 \times 2=2^5$

In this context, it makes sense to discuss the numerical values for 2^0 and 2^1.

3. With this new insight, have the students calculate how much confetti would be expected if the paper were folded 10 times: $2^{10}=1024$.

Discussion

- Ask your students to guess the minimum number of times they would need to fold the paper to get 1 million confetti. It would take only 20 folds, which is twice as many as that for just over a thousand. This is an example of how easy it is to underestimate the power of exponential growth.
- In this context, discuss with students the limits of confetti production due to the limitations of folding a sheet of paper.
- Note: the folding method for making confetti as quickly as possible is also used to make puff pastry with many layers. A nice idea would be to tie this into cooking lessons with a baking activity to make puff pastry.

Activity 2 – Exponential growth in a pandemic

In the previous example, the number of confetti doubled each time the paper was folded. What about the number of cases of a contagious disease? What does growth look like there?

Materials

- The accompanying worksheet
- A calculator
- A computer, tablet, or smartphone with an internet browser

Part 1 -

1. Let students read the background information on their worksheet about the R_0 and D values for COVID-19 infection.
2. Ask students to plot the number of newly infected people on the diagram with concentric circles. On the circles, which mark periods of five days, they should indicate each newly infected person as a dot. They should also draw connecting lines between newly infected people and the infecting person. The diagram should look like this:

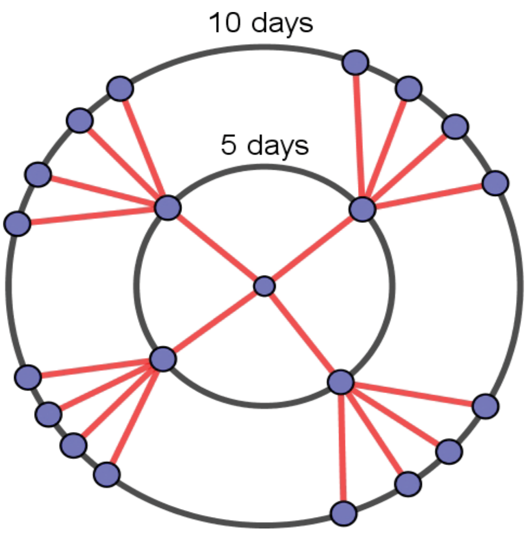


Image courtesy of Wolfgang Vieser

3. They should then fill in Table 2. It should look like this:

Days	0	5	10	15	20
Number of newly infected people	1	4	16	64	256

4. Students should then plot the results from the worksheet on the graph and connect the points in a curve. The graph should look like this:

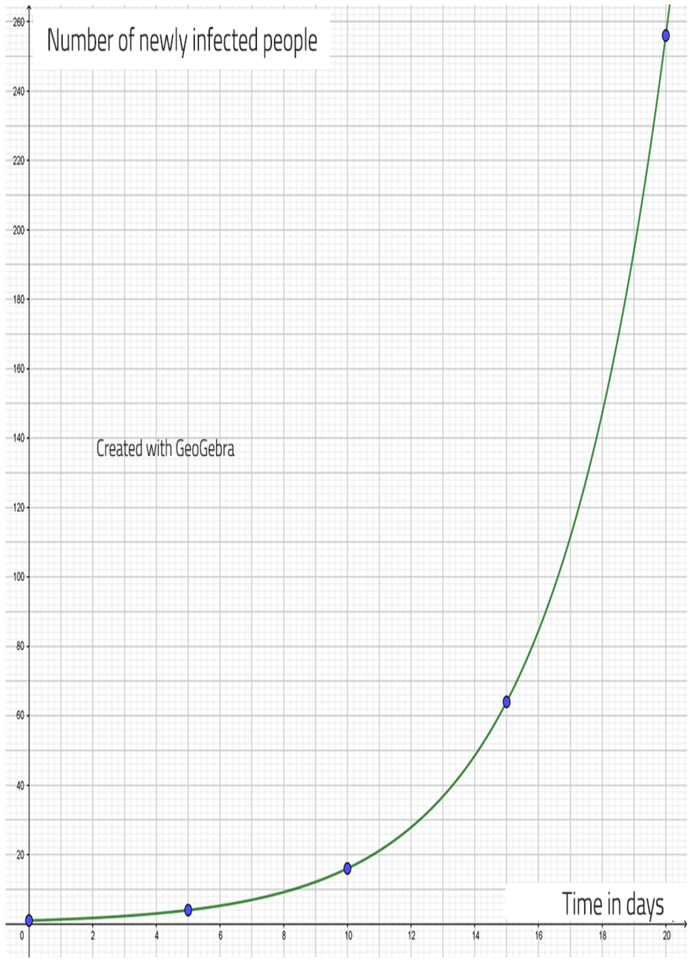


Image courtesy of Wolfgang Vieser

5. Have the students use the graph to determine how long it takes for the number of newly infected people to double. How long does it take for 4(→16→64) newly infected people to become 8(→32→128) newly infected people? The doubling time should be 2.5 days.
6. With a known doubling time, ask students to complete Table 3 on the worksheet. It should look like this:

Number of doublings	0	1	2	3	4	5	6	7	8
Days	0	2.5	5	7.5	10	12.5	15	17.5	20
Number of newly infected people	1	2	4	8	16	32	64	128	256

Discussion

- Have students compare Table 3 with Table 1 and recall how many paper folds were needed to make 1 million confetti. Discuss how many days would pass before 1 million people were newly infected. After approximately 20 doubling times, i.e., 20 × 2.5 days = 50 days, the number of newly infected individuals would exceed 1 million.
- Let the students guess the time required for the number

of newly infected to reach 7.8 billion people (the total global population) – it is about 82 days.

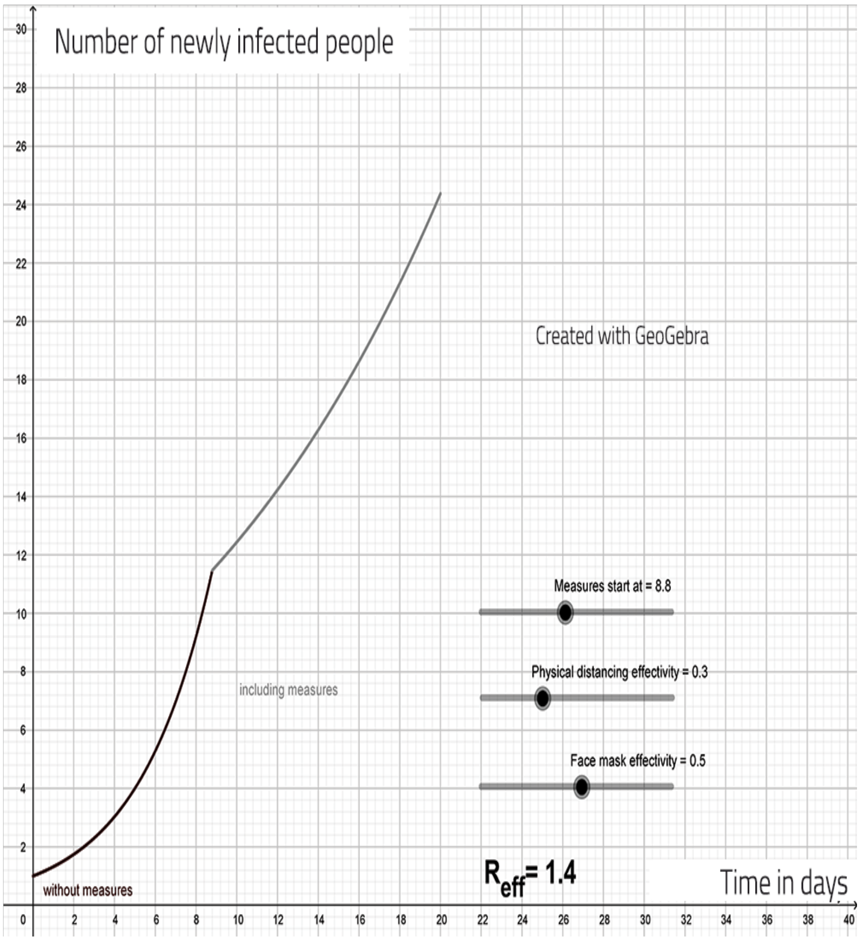
Part 2: Containment of the COVID-19 pandemic

1. Let the students discuss in groups which value, R_0 or *D*, could be changed by containment action (not vaccination or medication) and what those actions might be. It should become clear that *D* is not affected, but reducing R_0 through containment measures (physical distancing, face protection) could reduce the spread of the virus. This gives the 'effective reproduction number', *R*.
2. Discuss with your students the value of R required for the number of newly infected people to stop growing. The answer is an $R \leq 1$, i.e., an infected person can only infect a maximum of one other person.
3. Let the students read the background information on their worksheet about the connection between R_0 and *R* and have them use the Geogebra applet (https://www.geogebra.org/m/qavutkx5) to simulate different scenarios of COVID-19 containment. The Geogebra applet looks like this:
4. Have the students investigate the following questions:
   a. How does the timing of the onset of the containment measures change the course of the graph?
   b. How do the following containment measures affect the number of newly infected people?
      - physical distancing only
      - masks only
      - both physical distancing and masks

It should become clear that using the two measures together has the greatest effect on reducing new infections.

Discussion

Discuss which additional factors could also reduce the *R value*. They should see that the percentage of people who are not yet immune also affects the *R value*. If everyone has already been vaccinated or recovered from the disease, they can't be infected and the *R value* will drop to zero. An extended model that takes this into account can be found in the accompanying article for 14–16 year olds, Exponential growth 2: real-life lessons from the COVID-19 pandemic.



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Resources

- Watch a video on how repeatedly folding a piece of paper could get you all the way to the moon.
- See the concept of herd immunity demonstrated using mousetraps!

Institution

ESO

Author biography

Wolfgang Vieser is an astrophysicist and has worked for 14 years as a math and physics teacher at a secondary school. He works for ESO as Education Coordinator and is responsible for the educational program of ESO Supernova.