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Maths with fruit

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Have fun with fruit while helping your students to explore the concepts of area and volume, and learn more about their real-world applications.

It's often easy to estimate the area of a flat surface, but most things in the world aren't flat. The activities described in this article give students the opportunity to think about areas and volume in terms of some everyday, irregular-shaped items: fruit and vegetables.

The materials in these activities are easy to find and also very familiar, thus making the link between mathematical equations and situations encountered in everyday life. By using similar-shaped fruit and vegetables, we can show how linear dimensions, areas and volumes change in different ways as we scale up the structures.

The activities, which were presented at the Science on Stage Festival 2019, are very suitable for students aged 11–14 as an extension and application of the mathematics of geometric figures to everyday life. Activities 1 and 2 can also be used to introduce the topics of area and volume to younger pupils in a very concrete way.

Students can work in pairs for the activities, which can be completed in two one-hour classes.

Activity 1: Measuring the area of your hand

In this simple activity, students use squared paper to find the approximate surface area of an irregular object – the palm of their own hand.

Materials

Each pair of students will need:

- Some sheets of paper marked in 1 cm squares
- Two marker pens of different colours

Procedure

1. Start by asking students to guess the surface area in cm^2 (or number of squares) and record their guess. At the end of the activity, they can check back and see how close they were.
2. Ask students to place one hand on the graph paper, and use the other hand to draw the outline around it using one of the pens (figure 1).

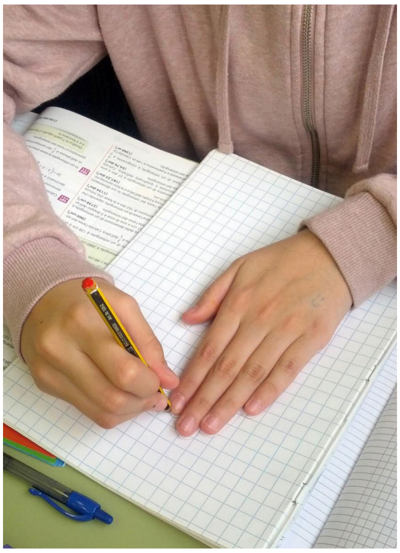


Figure 1: Drawing around the hand
Image courtesy of Maria Teresa Gallo

3. To find the area, each student should first count only the whole squares completely inside the outline.
4. Then they count all the whole squares that are wholly or just partly inside the outline. They can use a different pen colour to draw the outlines of the different counting methods (figure 2).
5. Find the average between the two counted areas (by adding the number together and dividing by 2). This number is quite close to the actual surface area of the palm.
6. Students can then compare this figure to the one they guessed at the start.

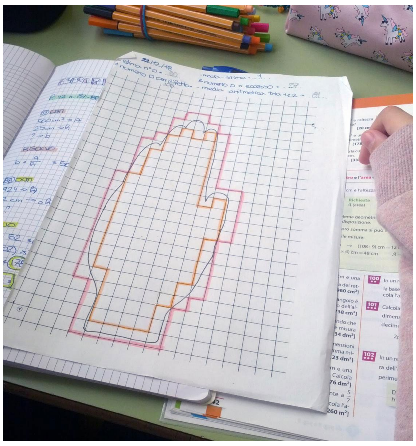


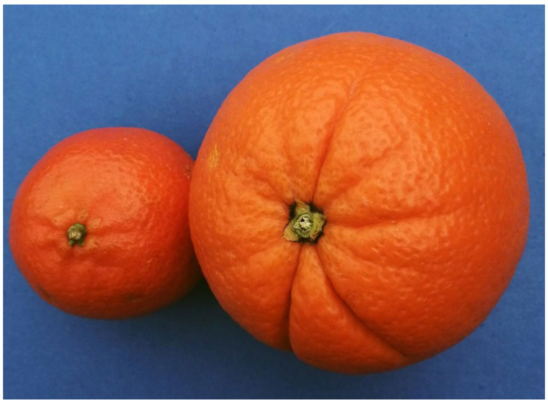
Figure 2: The hand outline, with the whole-squares and part-squares outlines
Image courtesy of Maria Teresa Gallo

Discussion

Use the following questions to review the students' results:

- Is there is much difference between the guesses at the start of the activity and the final figures obtained for the surface area of the palm?
- Which student made the best guess? You can have a little competition!

Activity 2: Measuring the surface area and volume of fruit



A clementine and an orange
Image courtesy of Maria Teresa Gallo

In this activity, students measure the diameter, surface area and volume of some more irregular objects: fruit and vegetables. This provides a basis for a more mathematical consideration of areas and volumes for older students in activity 3.

Materials

Each pair of students will need the following materials:

- Two easily peeled items of fruit or vegetables, with similar shapes but different sizes: one item should have a diameter approximately double that of the other (e.g. an orange and a clementine, a large and a small apple, or large and small potatoes of a similar shape)
- Peeler
- Some sheets of 1 cm squared paper
- Marker pen
- Tall plastic container
- Water
- Measuring jug
- Calliper (or a ruler)
- Optional: transparent food-wrap film (to cover the squared paper and keep it dry during the activities)

Procedure

Students should follow all the steps below. It's best to start with finding diameters and volumes as these can be measured with whole fruit, whereas the fruit need to be peeled to find the surface area, which reduces their volume.

Safety note

Warn students not to eat the fruit at the end of the activity, as it has not been prepared hygienically. You can provide a little bit of clean fruit to eat before or after the activity.

To find the diameters:

1. Measure the diameter of each item of fruit or vegetable using callipers (figure 3) and make a note.
2. If you have only a ruler, place the item between two cuboid-shaped objects (such as paperback books, tissue boxes or shoe boxes) arranged so that they are parallel and touching the item, then measure the distance between the objects.

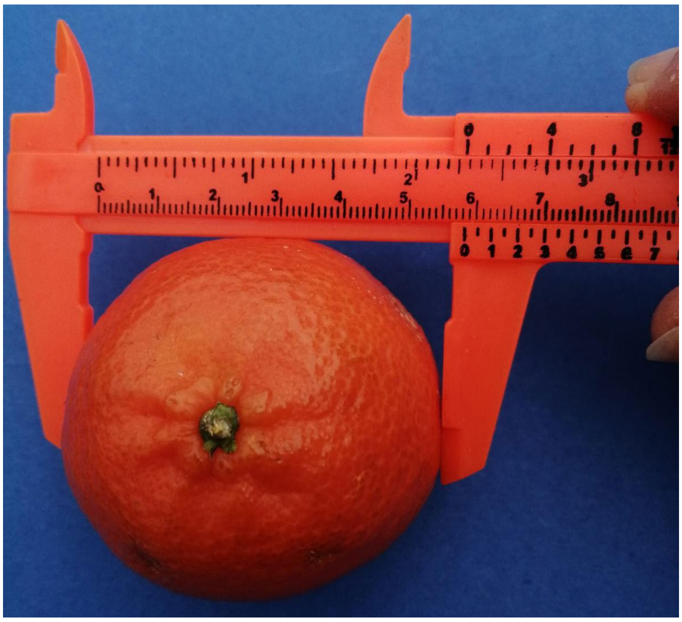


Figure 3: Measuring the diameter of a clementine
Image courtesy of Maria Teresa Gallo

To find the volumes:

1. Before peeling the fruit or vegetables, put one item in a tall container
2. Add enough water to cover the item, pushing it down with a fork (or pen) so that it is completely covered with water. Mark the water level reached (figure 4, left).
3. Remove the fruit, and mark the lower water level (figure 4, right).
4. Then put some water in the measuring jug and note the volume. Pour water from the measuring jug into the tall container to reach the higher level (with the fruit immersed) previously marked, and note the new, reduced volume in the measuring jug.
5. To find the volume of the fruit, subtract the second measuring jug volume from the first volume. This is the amount of water added to replace the volume of the fruit.
6. Do the same thing with the other (smaller or larger) fruit or vegetable item, recording the volume for each item.

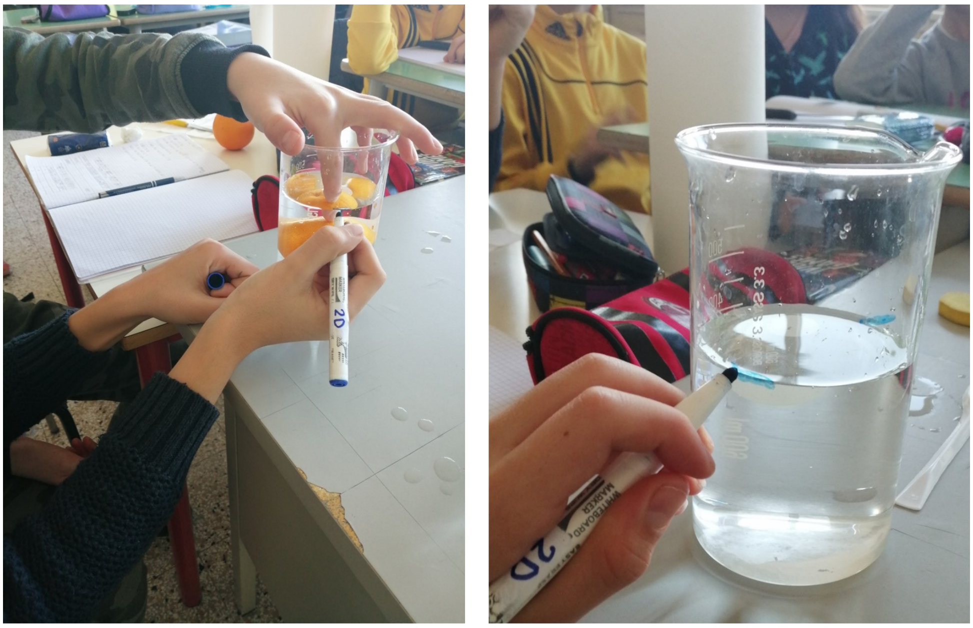


Figure 4: **Marking containers to measure fruit volumes**
Image courtesy of Maria Teresa Gallo

To find the surface area:

1. Peel the larger fruit or vegetable item very carefully, to obtain strips that are as long and as wide as possible.
2. Place all the strips of peel on the squared paper, placing the edges of the pieces as close as possible together to avoid empty spaces.
3. Using a marker pen, draw a close line around the shape created (figure 5).
4. Do the same thing with the smaller fruit or vegetable item. Be careful to keep all the peel pieces from one item separate from the other item.
5. Count the squares covered by the peel for each fruit or vegetable and record this number on the squared paper. This is the surface area, in cm^2.

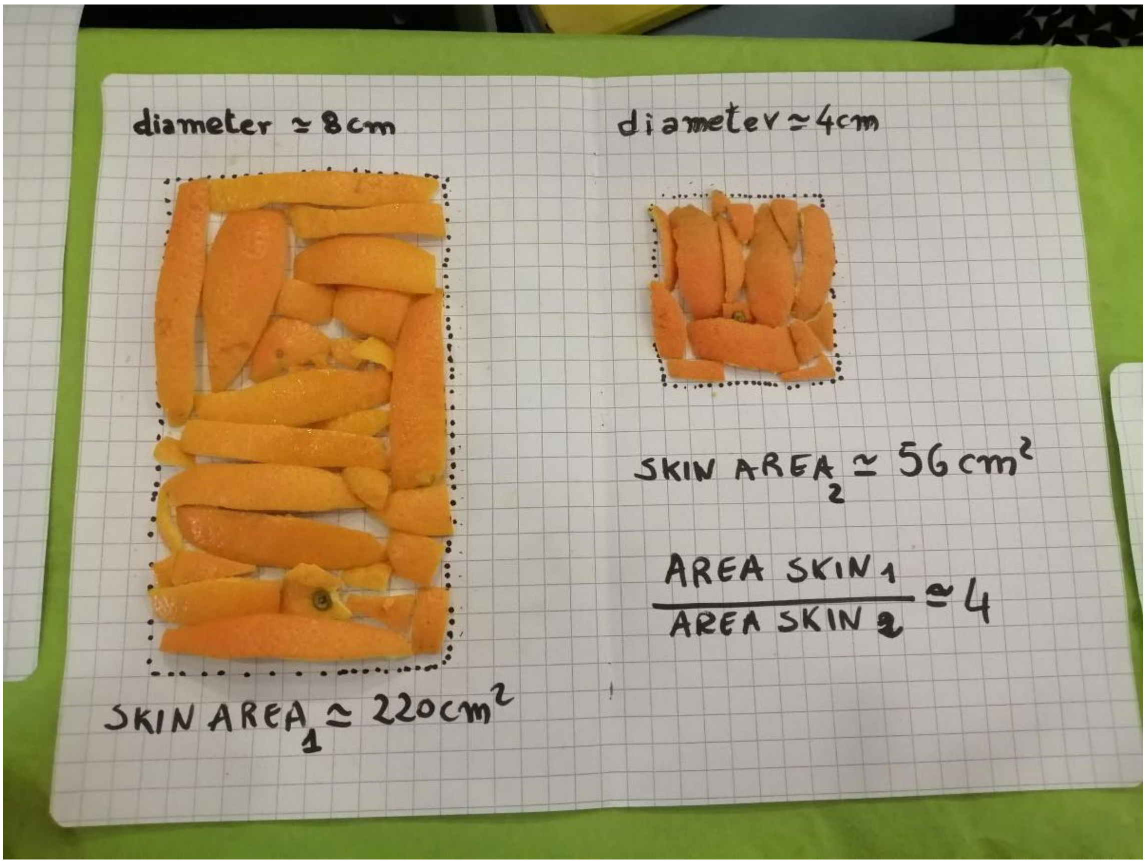


Figure 5: Finding the surface area of fruit by peeling and counting squares
Image courtesy of Maria Teresa Gallo

Results

Record all your measurements in a simple table, similar to table 1.

Object	Diameter (cm)	Area (cm^2)	Volume (cm^3)
Clementine (small fruit)	4	56	50
Orange (large fruit)	8	220	380

Table 1: Measurements of the diameter, surface area and volume of fruit

Discussion

In this activity, students discover one way of measuring the surface areas of objects (such as fruit) that aren't flat: peeling them. You can ask them to think of other ways of measuring areas, lengths and volumes of irregularly shaped objects. For example, they could make squares from masking tape and cover part of their body (e.g. their forearm) with the squares, to measure its area. Can they think of other examples?

Students can also begin to understand how areas and volumes change in relation to the length or diameter of the fruit. Ask them to consider: how does the surface area and volume of fruit double when you double the diameter (or other linear dimensions such as length or width)? From their measurements (table 1), it should be clear that doubling the diameter of a fruit increases the surface area and volume by factors that are much greater than double, even though the measurements are approximate.

Students will investigate this question further in the next activity.

Activity 3: Comparing the area and volume of fruit

In this activity, the students discover how an object's area and volume change when its linear measurements increase. Older students can also explore how the mathematical formulae for the surface area and volume of regular solids can be related to real, irregularly shaped objects such as fruit, if they are already familiar with these formulae (Part 2).

Materials

- Sets of interlocking 1 cm unit cubes (e.g. Regoliâ, MathLink Cubesâ)
- Graph paper
- Some fruit of different shapes (e.g., orange, apple, pear)
- For reference, mathematical formulae for surface areas of regular solids (cubes, spheres, cones, cylinders etc.)

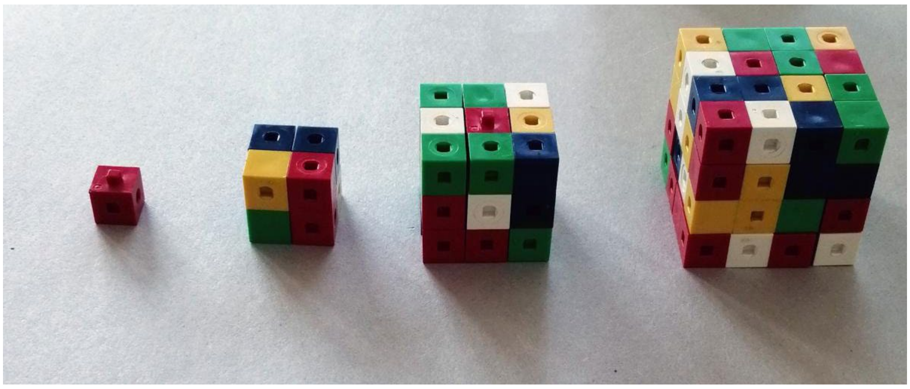


Figure 6: Cubes of increasing size made from unit cubes
Image courtesy of Maria Teresa Gallo

Procedure

Part 1: Unit cube modelling

1. Students should follow the steps below. They can work individually or in groups, depending on how many sets of unit cubes are available.
2. Using the unit cubes, build several cubes with sides of 1, 2, 3 and 4 units.
3. Count the surface area of each cube, and record this and the side length in a table (table 2)
4. Work out the volume using the formula l 3(l x l x l) where l is the side length, and enter this in the table
5. Now check this result by taking the cube apart and counting the number of cubes.
   Plot two graphs using the values in table 2: use the length of the cube side on the x axes, and the total surface area and volume on the y axes.
6. What do you notice about how (i) surface area and (ii) volume increase in relation to the side length?

Length of cube side (cm)	Area of cube base (cm^2)	Total surface area of cube (cm^2)	Volume of cube (cm^3)
1	1	6	1
2	4	24	8
3	9	54	27
4	16	96	64

Table 2: Linear, surface area and volume value for cubes of increasing size Part 2:

Fruit and regular solids calculations

1. Choose two fruit (e.g. orange and pear). Thinking about regular mathematical solids (e.g. spheres, cylinders, cones), work out how to model the approximate shape of your fruit by using combinations of different regular solids (figure 7).
2. Use the formulae for these solids to estimate the surface area and volume of the fruit. You can use the method in activity 2 to find the length and/or diameter of your fruit.

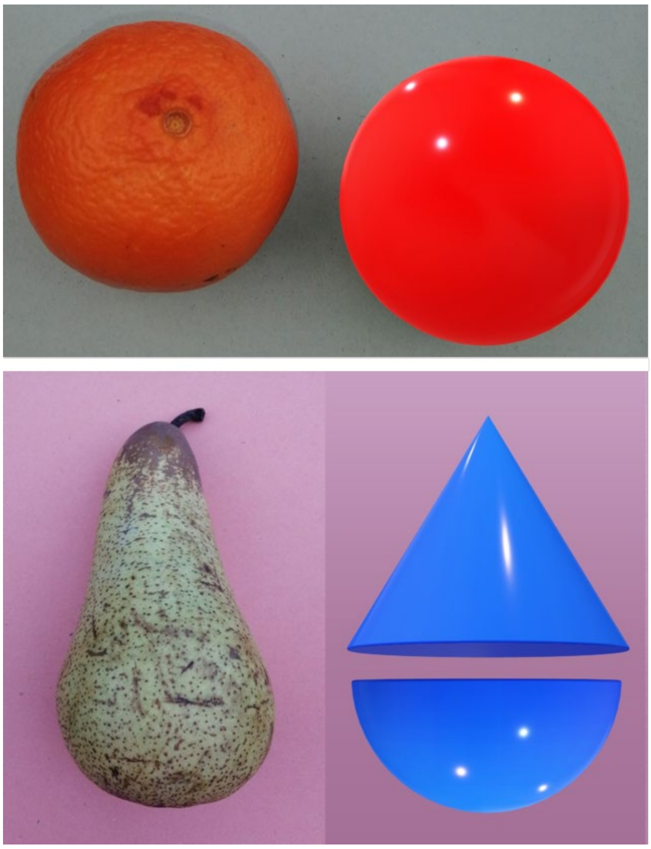


Figure 7: Approximating the irregular shape of fruit using regular solids
Image courtesy of Maria Teresa Gallo

Discussion

The following questions can be used to help students reflect on what they have learned in the activities:

- When the side length of a cube doubles, what happens to its surface area?
- When the side length of a cube doubles, what happens to its volume?
- Look back at your results from activity 2. Do you think the values you obtained approximately match these rules?
- How would the surface area and volume of a cube increase if its side length trebled?
- For cubes, what shape are the graphs of (i) surface area and (ii) volume against side length?What are the mathematical formulae for these shapes?

- How do you think surface area and volume increase with size for other shapes, e.g. spheres (in relation to their diameter) or cones (in relation to their base and height)?
- How closely do your calculated values match the real values of surface area and volume for irregularly shaped fruit from activity 2?

Looking at the table of values for cubes (table 2), students can verify that the area of one face of the cube (and the total surface area) grows according to the square of the side length (l^2), while the volume grows according to the cube of the side length (l^3), so volume grows faster than surface area in relation to increasing side length.

From the graphs, students can see that the plot for area against side length forms a parabola (quadratic relationship), while those for volume against side length form a cubic curve (cubic relationship).

Resources

- Find more information on Science on Stage: https://www.science-on-stage.eu/
- Discover a way to measure the surface area of your skin, and the pressure on it: www.exploratorium.edu/snacks/skin-size

Institution

Science on Stage

Author

Maria Teresa Gallo studied biology at the University of Palermo, Italy, and holds an MSc in science communication from S.I.S.S.A. in Trieste, Italy. She taught mathematics and science for many years in middle school. Now retired, she organizes science communication events for schools with the association Scienza under 18 Isontina, which she co-founded in 2010.

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