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How much carbon is locked in that tree?

Intercept theorem activity – how does the forester's triangle work?

In this activity, the students explore the mathematical theory behind the forester's triangle by employing a mathematical simulation. The teacher addresses the following question: how does the forester's triangle work? Note: an additional task for advanced students is included in the teacher's instructions.

This activity takes 30 minutes.

Materials

- Computers or laptops with internet access; ideally one for each group
- GeoGebra worksheet https://www.geogebra.org/m/md6fgpx5
- Intercept theorem worksheet

Procedure

1. Pose the question how does the forester's triangle work?
2. If necessary, briefly recap what students know about triangles, for example, the definitions of a right-angled triangle, an isosceles triangle, and a hypotenuse.
3. Open the GeoGebra (figure 1). In the beginning, don't use the slider "change forester's triangle".
4. Have the students work through the instructions for exercise 1 on the worksheet (or work through them as a class using a projector), moving the person in the GeoGebra worksheet back and forth to see how the values change.

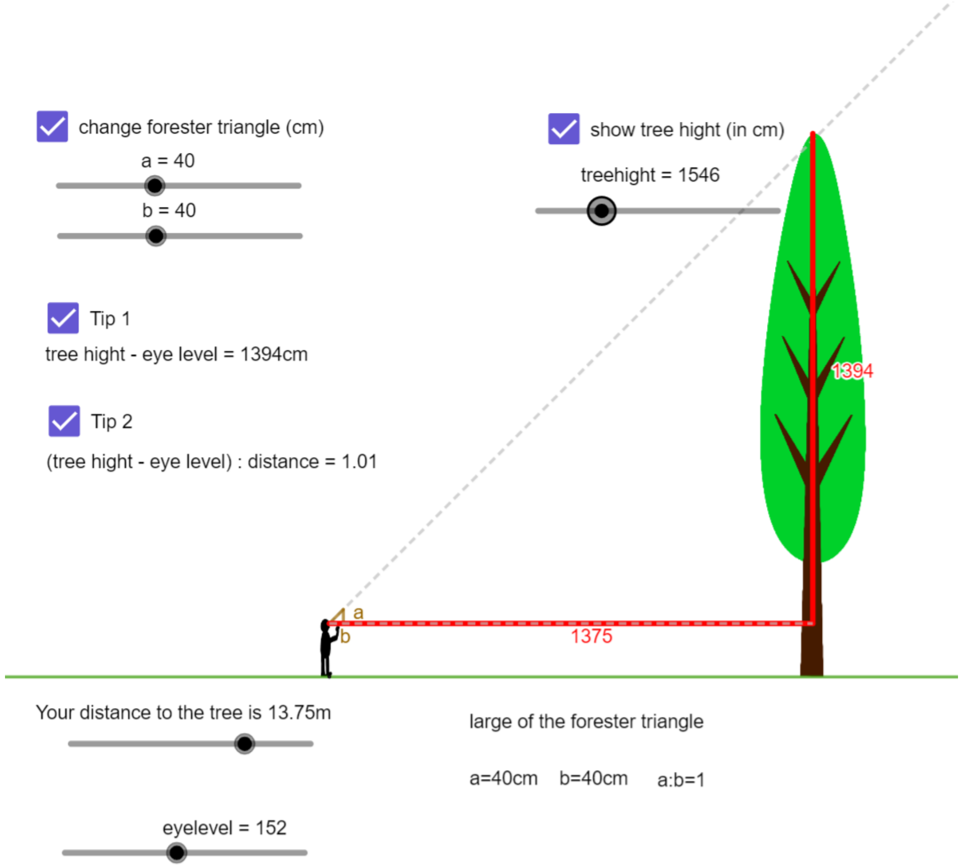

Figure 1: Screenshot of the worksheet on GeoGebra

Image: Michael Meyer, Körner/GeoGebra

5. The students should then be able to recognize a relationship for the distance to the tree when the hypotenuse lines up with the treetop. Hint: the height of the tree is measured from the ground and not from eye level (the horizontal line is coming from the person's eyes).
6. Have the students work through the instructions for exercise 2 on the worksheet. By changing the side lengths of the triangle, they should be able to deduce that the forester's triangle only works with isosceles, right-angled triangles.

Discussion

The intercept theorem is also known as Thales' theorem or the side-splitter theorem. It is one of the common theorems in elementary geometry. It states that if two rays start from one point and there are two parallel lines that intersect both rays, then certain ratios can be observed. The ratios that are important to see how the forester's triangle works are as follows (see figure 2):

$$\frac{|\overline{ZA}|}{|\overline{ZB}|} = \frac{|\overline{ZC}|}{|\overline{ZD}|} = \frac{|\overline{AC}|}{|\overline{BD}|}$$

For our tasks, $\overline{BD}$ describes the height of the tree (starting from the height of the eye to the top of the tree). $\overline{AC}$ describes one side of the triangle (side *a* in figure 1), and point *Z* is the forester's eye.

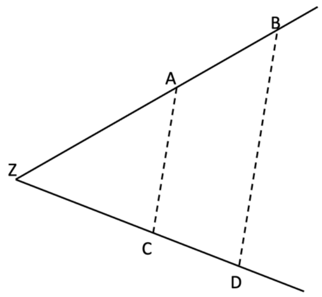

Figure 2: A graphic representation of intercept theorem

In the case of an isosceles forester's triangle (the sides $\overline{AC}$ and $\overline{ZC}$ are equal in length), we can reformulate the relationship:

$$\frac{|\overline{ZC}|}{|\overline{ZD}|} = \frac{|\overline{AC}|}{|\overline{BD}|}$$

$$\Leftrightarrow |\overline{BD}| = \frac{|\overline{ZD}|}{|\overline{ZC}|} \cdot |\overline{AC}|$$

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As $\frac{|\overline{AC}|}{|\overline{ZC}|} = 1$ (an isoceles forester's triangle is regarded), it follows:

$$|\overline{BD}| = |\overline{ZD}|$$

In task 1, the students should recognize that $|\overline{BD}| = |\overline{ZD}|$ for the isosceles triangle. They should therefore vary the distance of the forester from the tree. Activating tip 1 shows the height of the tree minus eye level.

Working with ratios is often very demanding for students. Therefore, task 2 can be used to make an intermediate step that is not very demanding (especially if tip 2 is used).

Task 3 describes the step to the intercept theorem, when the triangle is not an isosceles triangle. Here, the students should make the step from the application of the forester's triangle to the part of the mathematical intercept theorem. It would also be good to vary the distance of the forester to the tree in GeoGebra to make the intercept theorem recognizable.

The question of why the forester's triangle works cannot be answered by using this task. Other arguments must be used, for example, based on similarities. So the activity does not get out of hand (e.g., losing the connection to the biological questions and the application), this has not been realized here.